1999 AMC 8 Problems/Problem 1

Problem

$(6?3) + 4 - (2 - 1) = 5.$ To make this statement true, the question mark between the 6 and the 3 should be replaced by

$\text{(A)} \div \qquad \text{(B)}\ \times \qquad \text{(C)} + \qquad \text{(D)}\ - \qquad \text{(E)}\ \text{None of these}$

Solution

Simplify the given expression: $(6?3) + 4 - (2 - 1) = 5$

$(6?3) + 4 - 1 = 5$

$(6?3) + 3 = 5$

$(6?3) = 2$

At this point, it becomes clear that it should be $\div$, $\boxed{A}$.

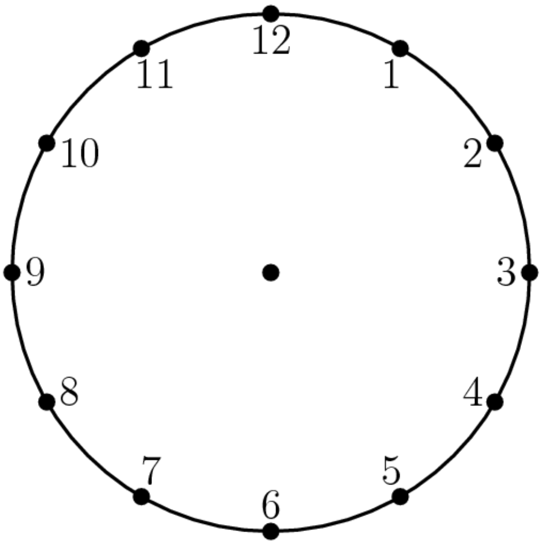
See Also

1999 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=1999))	
Preceded by First Question	Followed by Problem 2
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

1999 AMC 8 Problems/Problem 2

Problem

What is the degree measure of the smaller angle formed by the hands of a clock at 10 o'clock?



$\text{(A)}\ 30 \qquad \text{(B)}\ 45 \qquad \text{(C)}\ 60 \qquad \text{(D)}\ 75 \qquad \text{(E)}\ 90$

Solution

At $10:00$, the hour hand will be on the 10 while the minute hand on the 12.

This makes them $\frac{1}{6}$th of a circle apart, and $\frac{1}{6} \cdot 360^\circ = \boxed{60 \text{ (C)}}$.

See Also

1999 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=1999))	
Preceded by Problem 1	Followed by Problem 3
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

1999 AMC 8 Problems/Problem 3

Problem

Which triplet of numbers has a sum NOT equal to 1?

$\text{(A)}\ (1/2,1/3,1/6) \qquad \text{(B)}\ (2,-2,1) \qquad \text{(C)}\ (0.1,0.3,0.6) \qquad \text{(D)}\ (1.1,-2.1,1.0) \qquad \text{(E)}\ (-3/2,-5/2,5$

Solution

By adding each triplet, we can see that $\boxed{(D)}$ gives us 0, not 1, as our sum.

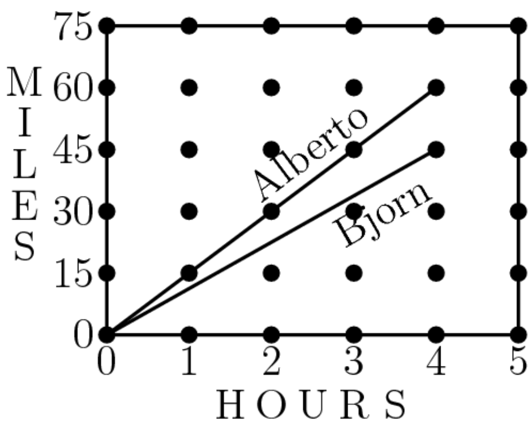
See Also

1999 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=1999))	
Preceded by Problem 2	Followed by Problem 4
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

1999 AMC 8 Problems/Problem 4

Problem

The diagram shows the miles traveled by bikers Alberto and Bjorn. After four hours, about how many more miles has Alberto biked than Bjorn?



$\text{(A)}\ 15 \qquad \text{(B)}\ 20 \qquad \text{(C)}\ 25 \qquad \text{(D)}\ 30 \qquad \text{(E)}\ 35$

Solution

After 4 hours, we see that Bjorn biked 45 miles, and Alberto biked 60. Thus the answer is $60-45=15$ $\boxed{\text{(A)}}$.

See Also

1999 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=1999))	
Preceded by Problem 3	Followed by Problem 5
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

1999 AMC 8 Problems/Problem 5

Problem

A rectangular garden 50 feet long and 10 feet wide is enclosed by a fence. To make the garden larger, while using the same fence, its shape is changed to a square. By how many square feet does this enlarge the garden?

$\text{(A)}\ 100 \qquad \text{(B)}\ 200 \qquad \text{(C)}\ 300 \qquad \text{(D)}\ 400 \qquad \text{(E)}\ 500$

Solution

The perimeter of the rectangular garden is $2(50+10)=120$ feet. A square with this perimeter has sidelength $120/4=30$ feet. The area of the rectangular garden is $(50)(10)=500$ and the area of the square garden is $(30)(30)=900$, so the area increases by $900-500=\boxed{\text{(D)}\ 400}$.

See Also

1999 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=1999))	
Preceded by Problem 4	Followed by Problem 6
1 • 2 • 3 • 4 • 5 • 6 • 7 • 8 • 9 • 10 • 11 • 12 • 13 • 14 • 15 • 16 • 17 • 18 • 19 • 20 • 21 • 22 • 23 • 24 • 25	
All AJHSME/AMC 8 Problems and Solutions	

1999 AMC 8 Problems/Problem 6

Problem

Bo, Coe, Flo, Jo, and Moe have different amounts of money. Neither Jo nor Bo has as much money as Flo. Both Bo and Coe have more than Moe. Jo has more than Moe, but less than Bo. Who has the least amount of money?

$\text{(A)}\ \text{Bo}\qquad\text{(B)}\ \text{Coe}\qquad\text{(C)}\ \text{Flo}\qquad\text{(D)}\ \text{Jo}\qquad\text{(E)}\ \text{Moe}$

Solution

Use logic to solve this problem. You don't actually need to use any equations.

Neither Jo nor Bo has as much money as Flo. So Flo clearly does not have the least amount of money. Rule out Flo.

Both Bo and Coe have more than Moe. Rule out Bo and Coe; they clearly do not have the least amount of money.

Jo has more than Moe. Rule out Jo.

The only person who has not been ruled out is Moe. So $\boxed{\text{(E)}}$ is the answer.

See Also

1999 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=1999))	
Preceded by Problem 5	Followed by Problem 7
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All AJHSME/AMC 8 Problems and Solutions	

1999 AMC 8 Problems/Problem 7

Problem

The third exit on a highway is located at milepost 40 and the tenth exit is at milepost 160. There is a service center on the highway located three-fourths of the way from the third exit to the tenth exit. At what milepost would you expect to find this service center?

$\text{(A)}\ 90 \qquad \text{(B)}\ 100 \qquad \text{(C)}\ 110 \qquad \text{(D)}\ 120 \qquad \text{(E)}\ 130$

Solution

There are $160-40=120$ miles between the third and tenth exits, so the service center is at milepost $40+(3/4)(120)=40+90=\boxed{\text{(E)}\ 130}$.

See Also

1999 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=1999))	
Preceded by Problem 6	Followed by Problem 8
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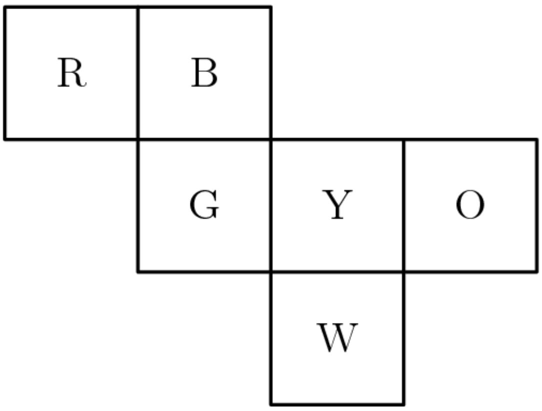
1999 AMC 8 Problems/Problem 8

Contents

Problem

Six squares are colored, front and back, (R = red, B = blue, O = orange, Y = yellow, G = green, and W = white). They are hinged together as shown, then folded to form a cube. The face opposite the white face is



$\text{(A)}\ \text{B} \qquad \text{(B)}\ \text{G} \qquad \text{(C)}\ \text{O} \qquad \text{(D)}\ \text{R} \qquad \text{(E)}\ \text{Y}$

Solution

Solution 1

When G is arranged to be the base, B is the back face and W is the front face. Thus, $\boxed{\text{(A)}\ B}$ is opposite W.

Solution 2

Let Y be the top and fold G, O, and W down. Then $\boxed{\text{(A)}\ B}$ will fold to become the back face and be opposite W.

See Also

1999 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=1999))	
Preceded by Problem 7	Followed by Problem 9
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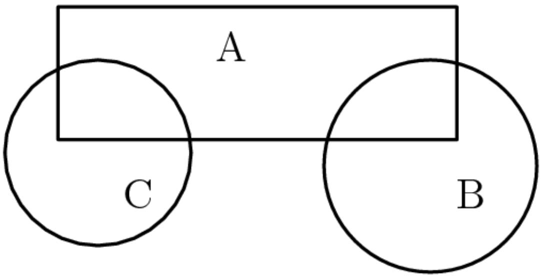
1999 AMC 8 Problems/Problem 9

Contents

Problem

Three flower beds overlap as shown. Bed A has 500 plants, bed B has 450 plants, and bed C has 350 plants. Beds A and B share 50 plants, while beds A and C share 100. The total number of plants is



$\text{(A)}\ 850 \qquad \text{(B)}\ 1000 \qquad \text{(C)}\ 1150 \qquad \text{(D)}\ 1300 \qquad \text{(E)}\ 1450$

Solution

Solution 1

Plants shared by two beds have been counted twice, so the total is $500+450+350-50-100=\boxed{\text{(C)}\ 1150}$.

Solution 2

Bed A has 350 plants it doesn't share with B or C. Bed B has 400 plants it doesn't share with A or C. And C has 250 it doesn't share with A or B. The total is $350+400+250+50+100=\boxed{\text{(C)}\ 1150}$ plants.

See Also

1999 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=1999))	
Preceded by Problem 8	Followed by Problem 10
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All AJHSME/AMC 8 Problems and Solutions	

1999 AMC 8 Problems/Problem 10

Contents

Problem

A complete cycle of a traffic light takes 60 seconds. During each cycle the light is green for 25 seconds, yellow for 5 seconds, and red for 30 seconds. At a randomly chosen time, what is the probability that the light will NOT be green?

$$\text{(A)}\ \frac{1}{4} \qquad \text{(B)}\ \frac{1}{3} \qquad \text{(C)}\ \frac{5}{12} \qquad \text{(D)}\ \frac{1}{2} \qquad \text{(E)}\ \frac{7}{12}$$

Solution

Solution 1

$$\frac{\text{time not green}}{\text{total time}} = \frac{R+Y}{R+Y+G} = \frac{35}{60} = \boxed{\text{(E)}\ \frac{7}{12}}$$

Solution 2

The probability of green is $\frac{25}{60}=\frac{5}{12}$, so the probability of not green is $1-\frac{5}{12}=\boxed{\text{(E)}\ \frac{7}{12}}$.

See Also

1999 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=1999))	
Preceded by Problem 9	Followed by Problem 11
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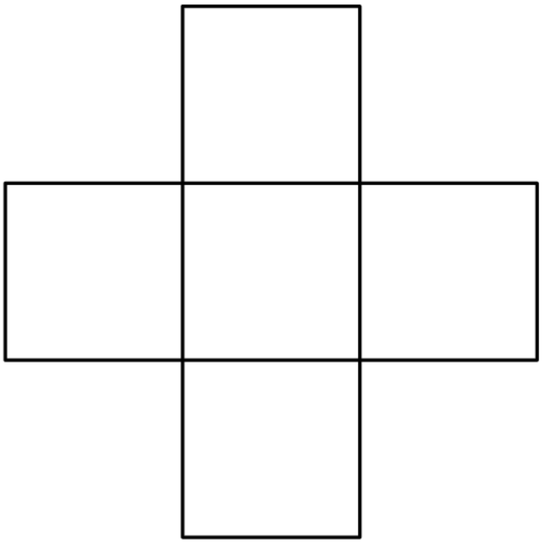
All AJHSME/AMC 8 Problems and Solutions	

1999 AMC 8 Problems/Problem 11

Contents

Problem

Each of the five numbers 1, 4, 7, 10, and 13 is placed in one of the five squares so that the sum of the three numbers in the horizontal row equals the sum of the three numbers in the vertical column. The largest possible value for the horizontal or vertical sum is



$\text{(A)}\ 20 \qquad \text{(B)}\ 21 \qquad \text{(C)}\ 22 \qquad \text{(D)}\ 24 \qquad \text{(E)}\ 30$

Solution

Solution 1

The largest sum occurs when 13 is placed in the center. This sum is $13 + 10 + 1 = 13 + 7 + 4 = \boxed{\text{(D)}\ 24}$. Note: Two other common sums, 18 and 21, are also possible.

Solution 2

Since the horizontal sum equals the vertical sum, twice this sum will be the sum of the five numbers plus the number in the center. When the center number is 13, the sum is the largest,

$$[10 + 4 + 1 + 7 + 2(13)] = 2S48 = 2SS = \boxed{\text{(D)}\ 24}$$

The other four numbers are divided into two pairs with equal sums.

See Also

1999 AMC 8 Problems/Problem 12

Problem

The ratio of the number of games won to the number of games lost (no ties) by the Middle School Middies is $11/4$. To the nearest whole percent, what percent of its games did the team lose?

$\text{(A)}\ 24\% \qquad \text{(B)}\ 27\% \qquad \text{(C)}\ 36\% \qquad \text{(D)}\ 45\% \qquad \text{(E)}\ 73\%$

Solution

The ratio means that for every 11 games won, 4 are lost, so the team has won $11x$ games, lost $4x$ games, and played $15x$ games for some positive integer x. The percentage of games lost is just
$\frac{4x}{15x}\times 100 = \frac{4}{15}\times 100 = 26.\overline{6}\% \approx \boxed{\text{(B)}\ 27\%}$

See also

1999 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=1999))	
Preceded by Problem 11	Followed by Problem 13
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All AJHSME/AMC 8 Problems and Solutions	

1999 AMC 8 Problems/Problem 13

Problem

The average age of the 40 members of a computer science camp is 17 years. There are 20 girls, 15 boys, and 5 adults. If the average age of the girls is 15 and the average age of the boys is 16, what is the average age of the adults?

$\text{(A)}\ 26 \qquad \text{(B)}\ 27 \qquad \text{(C)}\ 28 \qquad \text{(D)}\ 29 \qquad \text{(E)}\ 30$

Solution

First, find the total amount of the girl's ages and add it to the total amount of the boy's ages. It equals $(20)(15)+(15)(16)=540$. The total amount of everyone's ages can be found from the average age, $17\cdot 40=680$. Then you do $680-540=140$ to find the sum of the adult's ages. The average age of an adult is divided among the five of them, $140\div 5=\boxed{\text{(C)}\ 28}$.

See Also

1999 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=1999))	
Preceded by Problem 12	Followed by Problem 14
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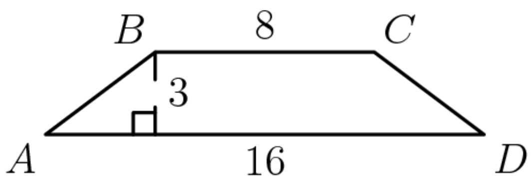
All AJHSME/AMC 8 Problems and Solutions	

1999 AMC 8 Problems/Problem 14

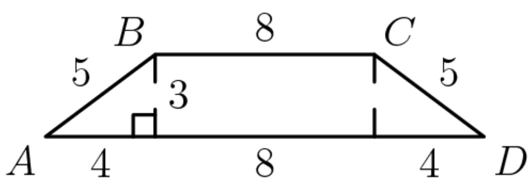
Problem

In trapezoid $ABCD$, the sides AB and CD are equal. The perimeter of $ABCD$ is



$\text{(A)}\ 27 \qquad \text{(B)}\ 30 \qquad \text{(C)}\ 32 \qquad \text{(D)}\ 34 \qquad \text{(E)}\ 48$

Solution



There is a rectangle present, with both horizontal bases being 8 units in length. The excess units on the bottom base must then be $16-8=8$. The fact that AB and CD are equal in length indicate, by the Pythagorean Theorem, that these excess lengths are equal. There are two with a total length of 8 units, so each is 4 units. The triangle has a hypotenuse of 5, because the triangles are $3-4-5$ right triangles. So, the sides of the trapezoid are 8, 5, 16, and 5. Adding those up gives us the perimeter, $8+5+16+5=13+21=\boxed{\text{(D)}\ 34}$ units.

See Also

1999 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=1999))	
Preceded by Problem 13	Followed by Problem 15
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1999 AMC 8 Problems/Problem 15

Contents

Problem

Bicycle license plates in Flatville each contain three letters. The first is chosen from the set $\{C, H, L, P, R\}$, the second from $\{A, I, O\}$, and the third from $\{D, M, N, T\}$.

When Flatville needed more license plates, they added two new letters. The new letters may both be added to one set or one letter may be added to one set and one to another set. What is the largest possible number of ADDITIONAL license plates that can be made by adding two letters?

$\text{(A)}\ 24 \qquad \text{(B)}\ 30 \qquad \text{(C)}\ 36 \qquad \text{(D)}\ 40 \qquad \text{(E)}\ 60$

Solution

Solution 1

There are currently 5 choices for the first letter, 3 choices for the second letter, and 4 choices for the third letter, for a total of $5 \cdot 3 \cdot 4 = 60$ license plates.

Adding 2 letters to the start gives $7 \cdot 3 \cdot 4 = 84$ plates.

Adding 2 letters to the middle gives $5 \cdot 5 \cdot 4 = 100$ plates.

Adding 2 letters to the end gives $5 \cdot 3 \cdot 6 = 90$ plates.

Adding a letter to the start and middle gives $6 \cdot 4 \cdot 4 = 96$ plates.

Adding a letter to the start and end gives $6 \cdot 3 \cdot 5 = 90$ plates.

Adding a letter to the middle and end gives $5 \cdot 4 \cdot 5 = 100$ plates.

You can get at most 100 license plates total, giving an additional $100 - 60 = 40$ plates, making the answer $\boxed{D}$

Solution 2

Using the same logic as above, the number of combinations of plates is simply the product of the size of each set of letters.

In general, when three numbers have the same fixed sum, their product will be maximal when they are as close together as possible. This is a 3D analogue of the fact that a rectangle with fixed perimeter maximizes its area when the sides are equal (ie when it becomes a square). In this case, no matter where you add the letters, there will be $5+3+4+2=14$ letters in total. If you divide them as evenly as possible among the three groups, you get $5, 5, 4$, which is a possible situation.

As before, the answer is $5 \cdot 5 \cdot 4 - 5 \cdot 3 \cdot 4 = 40$, and the correct choice is $\boxed{D}$

See Also

1999 AMC 8 Problems/Problem 16

Problem

Tori's mathematics test had 75 problems: 10 arithmetic, 30 algebra, and 35 geometry problems. Although she answered 70% of the arithmetic, 40% of the algebra, and 60% of the geometry problems correctly, she did not pass the test because she got less than 60% of the problems right. How many more problems would she have needed to answer correctly to earn a 60% passing grade?

$\text{(A)}\ 1 \qquad \text{(B)}\ 5 \qquad \text{(C)}\ 7 \qquad \text{(D)}\ 9 \qquad \text{(E)}\ 11$

Solution

First, calculate how many of each type of problem she got right:

Arithmetic: $70\% \cdot 10 = 0.70 \cdot 10 = 7$

Algebra: $40\% \cdot 30 = 0.40 \cdot 30 = 12$

Geometry: $60\% \cdot 35 = 0.60 \cdot 35 = 21$

Altogether, Tori answered $7 + 12 + 21 = 40$ questions correct. To get a 60% on her test overall, she needed to get $60\% \cdot 75 = 0.60 \cdot 75 = 45$ questions right.

Therefore, she needed to answer $45 - 40 = 5$ more questions to pass, and the the correct answer is $\boxed{B}$

See Also

1999 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=1999))	
Preceded by Problem 15	Followed by Problem 17
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1999 AMC 8 Problems/Problem 17

Problem

At Fat Papa Middle School the 108 students who take the Papa meet in the evening to talk about food and eat an average of two full size, double chocalate, creamy cream cakes apiece. Walter and Gretel are baking Bonnie's Smelliest Bar Cookies this year. Their recipe, which makes a pan of 15 cakes, lists this items: $1\frac{1}{2}$ cups flour, 2 eggs, 3 tablespoons butter, $\frac{3}{4}$ cups sugar, and 1 package of chocolate cakes. They will make only full recipes, not partial recipes.

Walter can buy eggs by the half-dozen. How many half-dozens should he buy to make enough cakes? (Some eggs and some cakes may be left over.)

$\text{(A)}\ 1 \qquad \text{(B)}\ 2 \qquad \text{(C)}\ 5 \qquad \text{(D)}\ 7 \qquad \text{(E)}\ 15$

Solution

If 108 students eat 2 cakes on average, there will need to be $108 \cdot 2 = 216$ cakes. There are 15 cakes per pan, meaning there needs to be $\frac{216}{15} = 14.4$ pans. However, since half-recipes are forbidden, we need to round up and make $\lceil\frac{216}{15}\rceil = 15$ pans.

1 pan requires 2 eggs, so 15 pans require $2 \cdot 15 = 30$ eggs. Since there are 6 eggs in a half dozen, we need $\frac{30}{6} = 5$ half-dozens of eggs, and the answer is $\boxed{C}$

See Also

1999 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=1999))	
Preceded by Problem 16	Followed by Problem 18
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1999 AMC 8 Problems/Problem 18

Contents

Problem

At Central Middle School the 108 students who take the AMC8 meet in the evening to talk about problems and eat an average of two cookies apiece. Walter and Gretel are baking Bonnie's Best Bar Cookies this year.

Their recipe, which makes a pan of 15 cookies, lists this items: $1\frac{1}{2}$ cups flour, 2 eggs, 3 tablespoons butter, $\frac{3}{4}$ cups sugar, and 1 package of chocolate drops. They will make only full recipes, not partial recipes.

They learn that a big concert is scheduled for the same night and attendance will be down 25%. How many recipes of cookies should they make for their smaller party?

$\text{(A)}\ 6 \qquad \text{(B)}\ 8 \qquad \text{(C)}\ 9 \qquad \text{(D)}\ 10 \qquad \text{(E)}\ 11$

Solution

Solution 1

If 108 students eat 2 cookies on average, there will need to be $108\cdot 2=216$ cookies. But with the smaller attendance, you will only need $100\%-25\%=75\%$ of these cookies, or $75\%\cdot 216=0.75\cdot 216=162$ cookies.

162 cookies requires $\frac{162}{15}=10.8$ batches. However, since half-batches are forbidden, we must round up to get $\lceil\frac{162}{15}\rceil=11$ batches, and the correct answer is $\boxed{E}$.

Solution 2

If there were 108 students before, with the 25% of students missing, there will be 75% of 108 students left. This is $75\%\cdot 108=0.75\cdot 108=81$ students. These students eat $81\cdot 2=162$ cookies. Follow the logic of the second paragraph above to find that there needs to be 11 batches, and the correct answer is $\boxed{E}$.

See Also

1999 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=1999))	
Preceded by Problem 17	Followed by Problem 19
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1999 AMC 8 Problems/Problem 19

Problem

At Central Middle School the 108 students who take the AMC 8 meet in the evening to talk about problems and eat an average of two cookies apiece. Walter and Gretel are baking Bonnie's Best Bar Cookeis this year. Their recipe, which makes a pan of 15 cookies, lists these items: $1\frac{1}{2}$ cups flour, 2 eggs, 3 tablespoons butter, $\frac{3}{4}$ cups sugar, and 1 package of chocolate drops They will only make full recipes, not partial recipes.

The drummer gets sick. The concert is cancelled. Walter and Gretel must make enough pans of cookies to supply 216 cookies. There are 8 tablespoons in a stick of butter. How many sticks of butter will be needed? (Some butter may be left over, of course.)

$\text{(A)}\ 5 \qquad \text{(B)}\ 6 \qquad \text{(C)}\ 7 \qquad \text{(D)}\ 8 \qquad \text{(E)}\ 9$

Solution

For 216 cakes, you need to make $\frac{216}{15} = 14.4$ pans. Since fractional pans are forbidden, round up to make $\lceil \frac{216}{15} \rceil = 15$ pans.

There are 3 tablespoons of butter per pan, meaning $3 \cdot 15 = 45$ tablespoons of butter is required for 15 pans.

Each stick of butter has 8 tablespoons, so we need $\frac{45}{8} = 5.625$ sticks of butter. However, we must round up again because partial sticks of butter are forbidden since they will be eaten in one gulp. Thus, we need $\lceil \frac{45}{8} \rceil = 6$ sticks of butter, and the answer is $\boxed{B}$.

See Also

1999 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=1999))	
Preceded by Problem 18	Followed by Problem 20
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1999 AMC 8 Problems/Problem 20

Problem

Figure 1 is called a "stack map." The numbers tell how many cubes are stacked in each position. Fig. 2 shows these cubes, and Fig. 3 shows the view of the stacked cubes as seen from the front.

Which of the following is the front view for the stack map in Fig. 4?

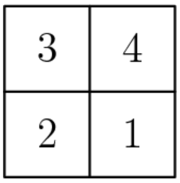


Figure 1

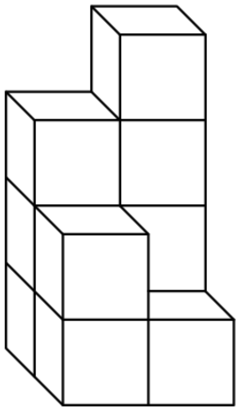


Figure 2

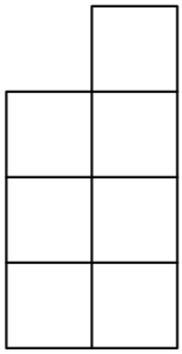


Figure 3

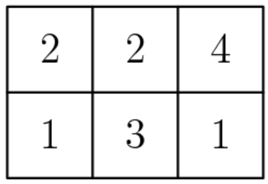
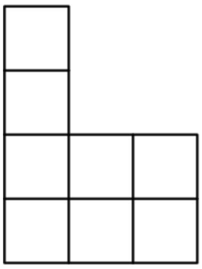
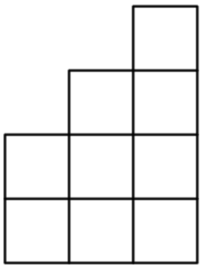
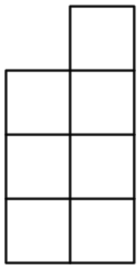
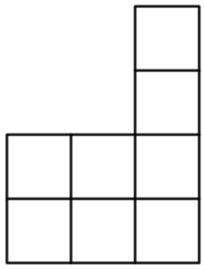


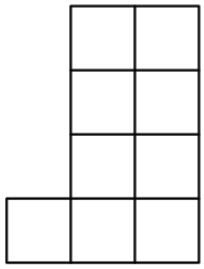
Figure 4


(A)


(B)


(C)


(D)


(E)

Solution

The third view is a direct, head-on view of the cubes. Thus, you will only see the highest (or, in these cases, higher) tower in each up-down column. For figure 4:

The highest tower in the first up-down column is 2 in the upper-left box.

The highest tower in the second up-down column is the 3 in the lower-middle box.

The highest tower in the third up-down column is the 4 in the upper-right box.

Thus, the head-on view of this tower should have 2 boxes on the left, 3 in the middle, and 4 on the right.

Diagram $\boxed{B}$ shows this description.

As trivia, option C shows the stack from the point of view of an observer on the right, facing towards the left.

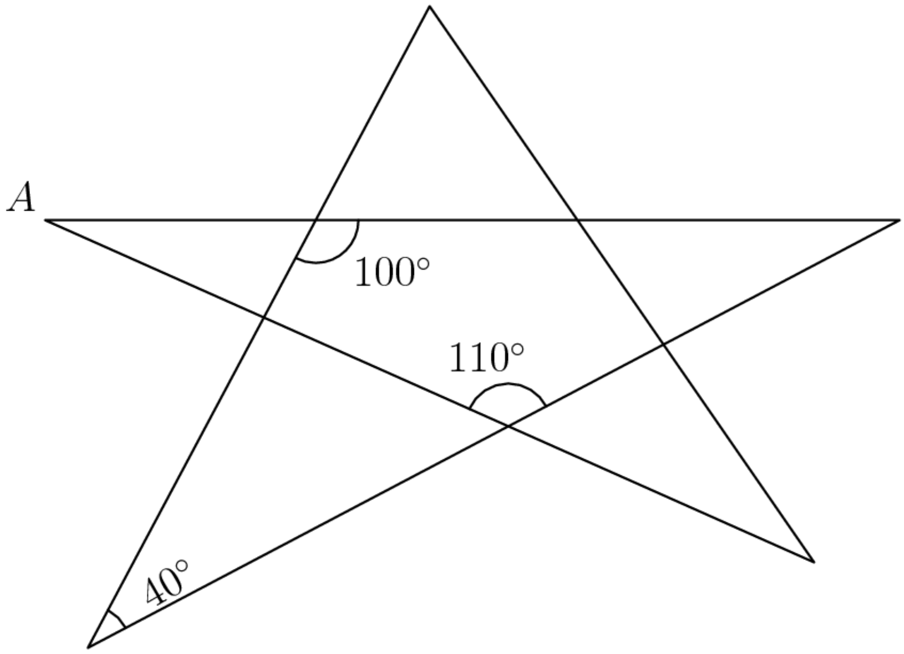
See Also

1999 AMC 8 Problems/Problem 21

Contents

Problem

The degree measure of angle A is



$\text{(A)}\ 20 \qquad \text{(B)}\ 30 \qquad \text{(C)}\ 35 \qquad \text{(D)}\ 40 \qquad \text{(E)}\ 45$

Solution

Solution 1

Angle-chasing using the small triangles:

Use the line below and to the left of the 110° angle to find that the rightmost angle in the small lower-left triangle is $180 - 110 = 70^\circ$.

Then use the small lower-left triangle to find that the remaining angle in that triangle is $180 - 70 - 40 = 70^\circ$.

Use congruent vertical angles to find that the lower angle in the smallest triangle containing A is also 70°.

Next, use line segment AB to find that the other angle in the smallest triangle containing A is $180 - 100 = 80^\circ$.

The small triangle containing A has a 70° angle and an 80° angle. The remaining angle must be $180 - 70 - 80 = \boxed{30^\circ, B}$

Solution 2

The third angle of the triangle containing the 100° angle and the 40° angle is $180^\circ - 100^\circ - 40^\circ = 40^\circ$. It follows that A is the third angle of the triangle consisting of the found 40° angle and the given 110° angle. Thus, A is a $180^\circ - 110^\circ - 40^\circ = 30^\circ$ angle, and so the answer is $\boxed{30^\circ, \textbf{B}}$.

See Also

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1999 AMC 8 Problems/Problem 22

Problem

In a far-off land three fish can be traded for two loaves of bread and a loaf of bread can be traded for four bags of rice. How many bags of rice is one fish worth?

$\text{(A)}\ \frac{3}{8} \qquad \text{(B)}\ \frac{1}{2} \qquad \text{(C)}\ \frac{3}{4} \qquad \text{(D)}\ 2\frac{2}{3} \qquad \text{(E)}\ 3\frac{1}{3}$

Solution

Let f represent one fish, l a loaf of bread, and r a bag of rice. Then: $3f = 2l,\ l = 4r$

Substituting l from the second equation back into the first gives us $3f = 8r$. So each fish is worth $\frac{8}{3}$ bags of rice, or $2\frac{2}{3} \Rightarrow \boxed{D}$.

See Also

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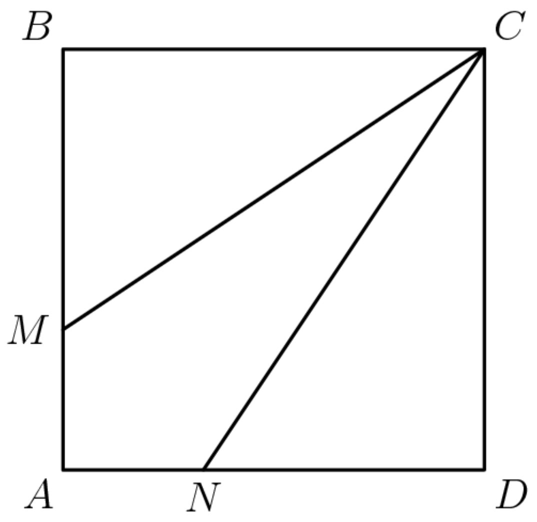
1999 AMC 8 Problems/Problem 23

Problem

Square $ABCD$ has sides of length 3. Segments CM and CN divide the square's area into three equal parts. How long is segment CM?



$\text{(A)}\ \sqrt{10} \qquad \text{(B)}\ \sqrt{12} \qquad \text{(C)}\ \sqrt{13} \qquad \text{(D)}\ \sqrt{14} \qquad \text{(E)}\ \sqrt{15}$

Solution

Since the square has side length 3, the area of the entire square is 9.

The segments divide the square into 3 equal parts, so the area of each part is $9 \div 3 = 3$.

Since $\triangle CBM$ has area 3 and base $CB = 3$, using the area formula for a triangle:

$$A_{tri} = \frac{1}{2}bh$$

$$3 = \frac{1}{2}3h$$

$$h = 2$$

Thus, height $BM = 2$.

Since $\triangle CBM$ is a right triangle, $CM = \sqrt{BM^2 + BC^2} = \sqrt{2^2 + 3^2} = \boxed{\text{(C)}\ \sqrt{13}}$.

See Also

1999 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=1999))	
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1999 AMC 8 Problems/Problem 24

Contents

Problem

When 1999^{2000} is divided by 5, the remainder is

$\text{(A)}\ 4 \qquad \text{(B)}\ 3 \qquad \text{(C)}\ 2 \qquad \text{(D)}\ 1 \qquad \text{(E)}\ 0$

Solution 1

Note that the units digits of the powers of 9 have a pattern: $9^1=\mathbf{9}, 9^2=8\mathbf{1}, 9^3=72\mathbf{9}, 9^4=656\mathbf{1}$, and so on. Since all natural numbers with the same last digit have the same remainder when divided by 5, the entire number doesn't matter, just the last digit. For even powers of 9, the number ends in a 1. Since the exponent is even, the final digit is 1. Note that all natural numbers that end in 1 have a remainder of 1 when divided by 5. So, our answer is $\boxed{\text{(D)}\ 1}$.

Solution 2

Write 1999 as $2000-1$. We are taking $(2000-1)^{2000} \mod 10$. Using the binomial theorem, we see that ALL terms in this expansion are divisible by 2000 except for the very last term, which is just $(-1)^{2000}$. This is clear because the binomial expansion is just choosing how many 2000s and how many -1s there are for each term. Using this, we can take the entire polynomial $\mod 10$, which leaves just $(-1)^{2000} = \boxed{\text{(D)}\ 1}$.

Solution 3

As $1999 \equiv -1 \pmod{5}$, we have $1999^{2000} \equiv (-1)^{2000} \equiv 1 \pmod{5}$. Thus, the answer is $\boxed{\text{(D)}\ 1}$.

See Also

1999 AMC 8 (Problems • Answer Key • Resources (http://www.artofproblemsolving.com/Forum/resources.php?c=182&cid=42&year=1999))	
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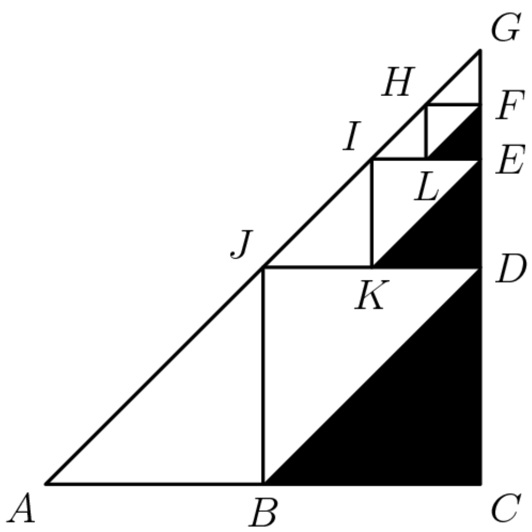
1999 AMC 8 Problems/Problem 25

Contents

Problem

Points B, D, and J are midpoints of the sides of right triangle ACG. Points K, E, I are midpoints of the sides of triangle JDG, etc. If the dividing and shading process is done 100 times (the first three are shown) and $AC = CG = 6$, then the total area of the shaded triangles is nearest



$\text{(A)}\ 6 \qquad \text{(B)}\ 7 \qquad \text{(C)}\ 8 \qquad \text{(D)}\ 9 \qquad \text{(E)}\ 10$

Solution

Solution 1

Since $\triangle FGH$ is fairly small relative to the rest of the diagram, we can make an underestimate by using the current diagram. All triangles are right-isosceles triangles.

$$CD = \frac{CG}{2} = 3, DE = \frac{CD}{2} = \frac{3}{2}, EF = \frac{DE}{2} = \frac{3}{4}$$

$$CB = CD = 3, DK = DE = \frac{3}{2}, EL = EF = \frac{3}{4}$$

$$[CBD] = \frac{1}{2}3^2 = \frac{9}{2}$$

$$[DKE] = \frac{1}{2}(\frac{3}{2})^2 = \frac{9}{8}$$

$$[ELF] = \frac{1}{2}(\frac{3}{4})^2 = \frac{9}{32}$$

The sum of the shaded regions is $\frac{9}{2} + \frac{9}{8} + \frac{9}{32} = \frac{189}{32} \approx 5.9$

5.9 is an underestimate, as some portion (but not all) of $\triangle FGH$ will be shaded in future iterations.

If you shade all of $\triangle FGH$, this will add an additional $\frac{9}{32}$ to the area, giving $\frac{198}{32} \approx 6.2$, which is an overestimate.

Thus, $6 \rightarrow \boxed{A}$ is the only answer that is both over the underestimate and under the overestimate.

Solution 2

In iteration 1, congruent triangles $\triangle ABJ, \triangle BDJ,$ and $\triangle BDC$ are created, with one of them being shaded.

In iteration 2, three more congruent triangles are created, with one of them being shaded.

As the process continues indefnitely, in each row, $\frac{1}{3}$ of each triplet of new congruent triangles will be shaded. The "fourth triangle" at the top ($\triangle FGH$ in the diagram) will gradually shrink, leaving about $\frac{1}{3}$ of the area shaded. This means $\frac{1}{3}\left(\frac{1}{2}6 \cdot 6\right) = 6$ square units will be shaded when the process goes on indefinitely, giving $\boxed{A}$.

Solution 3

Using Solution 1 as a template, note that the sum of the areas forms a geometric series:

$$\frac{9}{2} + \frac{9}{8} + \frac{9}{32} + \frac{9}{128} + ...$$

This is the sum of a geometric series with first term $a_1 = \frac{9}{2}$ and common ratio $r = \frac{1}{4}$

The sum of an infinite geometric series with $|r| < 1$ is $S_{\infty} = \frac{a_1}{1-r} = \frac{\frac{9}{2}}{1-\frac{1}{4}} = \frac{9}{2} \cdot \frac{4}{3} = 6,$ giving an answer of $\boxed{A}$.

See Also

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